

A Rate Equation Approach to Gain Saturation Effects
in Laser Mode Calculations

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Abstract

Space exploration and research will require large amounts of power. Solar pumped laser systems have been shown to have potential for meeting the performance requirements necessary for power transmission in space. The successful design of laser systems for these applications will depend, in part, on having a clear understanding of the development of the dynamical processes in the laser cavity and on the effects that changes in physical and design parameters have on laser performance. In particular, it is necessary to know the amplitude and phase distributions of the laser beam at the output aperture when steady state operation is achieved in order to determine the far-field power distribution. The output from the laser will depend on the active medium, the optical environment of the active material and on the gain distribution in the active region as laser action builds up and reaches steady state.

An important component in the design process will be a realistic model of the active laser cavity. A computer model of the laser cavity, based on Huygens' principle, has been developed by M. D. Williams in the Space Systems Division/ High Energy Science Branch. The code calculates the amplitude and phase of an optical wave reflected back and forth between the mirrors of a laser cavity. The original code assumes a gain distribution which does not change with the build up of oscillations in the cavity. A step in the direction of realism is the inclusion of a saturable gain medium in the cavity. The objective of this study is to incorporate saturation effects into the existing computer model.

As an optical wave propagates back and forth between the mirrors of an optical resonator, it satisfies the Fresnel-Kirchhoff formulation of Huygens' principle which is summarized briefly below. If $U(x_1, y_1)$ is the complex amplitude field at a point (x_1, y_1) on a mirror M_1 , then the field V at a point (x_2, y_2) on mirror M_2 can be determined by the following propagation equation:

$$V(x_2, y_2) = \exp(-jkD) \int_{M_1} K(x_2, y_2; x_1, y_1) U(x_1, y_1) dS \quad (1)$$

where D is the distance between the mirrors and K is the propagation kernel which depends on the aperture and other intervening optical elements in the cavity. The integral is a surface integral which is evaluated over the aperture.

A transverse mode represents a field which reproduces its structure after a passage through the resonator and hence is an eigenfunction of the integral operator defined by (1). The method of solution of the eigenvalue problem, first described by Fox and Li in 1961, involves an iterative procedure in which the wave is propagated back and forth until a steady state is achieved.

In the case of a saturable gain medium in which the gain has variations transverse to the optical axis, the integral equation is

$$V(x_2, y_2) = \exp(-jkD) \int_{M_1} \exp(\alpha D) K(x_2, y_2; x_1, y_1) U(x_1, y_1) dS \quad (2)$$

where α , which depends on the transverse position, is the complex amplitude gain coefficient. The gain coefficient is proportional to the population inversion density in the active material and is given by $\alpha = \sigma N/2$. The parameter σ is the stimulated emission cross section and N is the population inversion density.

The theoretical study of transient laser dynamics frequently involves the use of rate equations which describe the temporal evolution of the excited states and photon density in the laser cavity. The number and complexity of the rate equations depends primarily on the nature of the active material. In this study, an idealized four-level model was used, but the principles involved are general enough to be used for other systems. It is assumed that there are no axial variations in either the population inversion density or the photon density. For simplicity, a uniform and continuous pumping rate is assumed. In laser mode calculations, the usual approach is to assume that the active medium is concentrated into two thin sheets which are adjacent to the mirrors. The optical wave is propagated through a free space length and the population inversion density is calculated by integrating the rate equations over the time required for the optical wave to pass through the length of the cavity. Then the field is multiplied by the gain and appropriate mirror reflection coefficient. The surface integral (2) is then computed. Iteration continues until a steady state is achieved.

Steady state conditions in a laser system are reached in times on the order of microseconds, but the computer calculations are very intensive and time-consuming. Thousands of calculations are performed to compute the surface integral, and the integration of the laser rate equations, accomplished by a fixed step Runge-Kutta algorithm, requires 5 evaluations of the derivatives per time step. An obvious shortcoming to this approach is the time expense for computation.

For all calculations, physical parameters typical of solid state lasers were used and pumping rates were well above threshold. The procedure worked well for the case of plane parallel mirrors, both with rectangular and circular apertures. The steady state profiles at the output aperture of the normalized population inversion, relative intensity, and phase, respectively, for a plane parallel resonator are shown in Fig. 1. For the confocal resonator, however, very small time increments were necessary to integrate the rate equations, and the computation is very sensitive to the magnitude of the pumping rate. Work has begun to incorporate an adaptive integration algorithm which will speed up the integration and prevent numerical overflow produced by accumulated truncation error.

References

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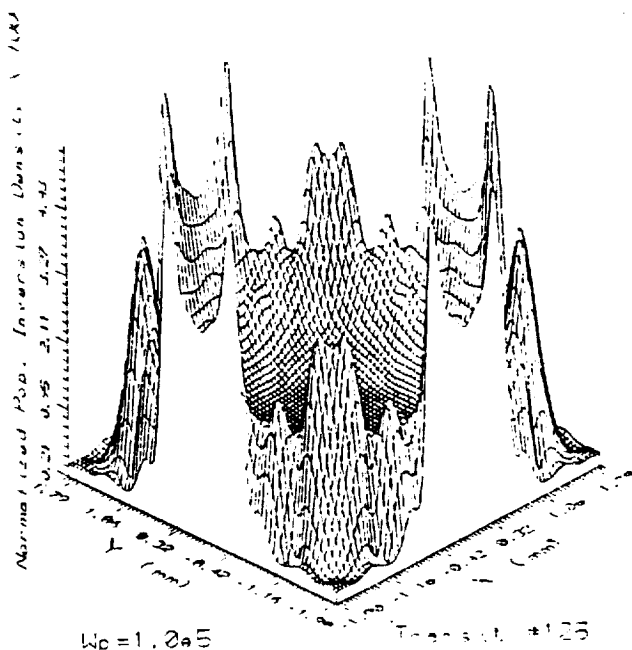


Figure 1. Population Inversion Density, Normalized Intensity, and Phase at the output mirror after 125 transits of the initial optical field through a plane-parallel resonator with circular aperture.

